(19) 7/12/2018

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	: **102**	**I**
Unique Paper Code	: **32221101**	
Name of Paper	: **Mathematical Physics - I**	
Name of Course	: **B.Sc. (Hons.) Physics**	
Semester	: **I**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all.
Question No. 1 is compulsory.

1. Do any *five* questions :

(a) Solve: $\frac{dy}{dx} = (1+x^2)(1+y^2)$.

(b) By calculating the Wronskian of the functions e^x, e^{-x}, and e^{-2x} check whether the functions are linearly dependent or independent.

(c) Find the area of the triangle with vertices P(2, 3, 5), Q(4, 2–1), and R(3, 6, 4).

(d) Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 4$ at the point $(1, \sqrt{2}, -1)$.

(e) Show that :

$$\oiint_S \left(\vec{\nabla} r^2\right) \cdot \overrightarrow{dS} = 6V$$

where S is the closed surface enclosing the volume V.

P.T.O.

(f) Evaluate :

$$\iint\limits_R \sqrt{x^2+y^2}\,dx\,dy$$

(g) Verify that :

$$\int_{-\infty}^{+\infty} \delta(a-x)\delta(x-b)\,dx = \delta(a-b)$$

(h) Form a differential equation whose solutions are e^{2x} and e^{3x}. 5×3=15

2. (a) Solve the inexact equation :

$$y(1+xy)\,dx + x(1+xy+x^2y^2)\,dy = 0.$$ 5

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x.$$ 4

(c) Using method of undetermined coefficients, solve the differential equation :

$$\frac{d^2y}{dx^2} + 4y = 2\sin 2x.$$ 6

3. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = x^2 + 5.$$ 9

(b) Solve the differential equation using method of variation of parameter

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 e^{2x}.$$ 6

4. (a) Show that

$$\left[\left(\vec{A}\times\vec{B}\right)\times\vec{C}\right]\times\vec{D} + \left[\left(\vec{B}\times\vec{A}\right)\times\vec{D}\right]\times\vec{C} + \left[\left(\vec{C}+\vec{D}\right)\times\vec{A}\right]\times\vec{B} + \left[\left(\vec{D}\times\vec{C}\right)\times\vec{B}\right]\times\vec{A} = 0.$$ 6

(b) Show that :

$$\vec{F} = \left(y^2\cos x + z^3\right)\hat{i} + \left(2y\sin x - 4\right)\hat{j} + \left(3xz^2+2\right)\hat{k}$$

is a conservative force field and then evaluate

$$\int_C \vec{F}\cdot\overrightarrow{dr}$$

where C is any path from (0, 1, –1) to $\left(\frac{\pi}{2}, -1, 2\right)$. 9

5. (a) If $\vec{a}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then prove that :

$$\text{curl}\left(\frac{\vec{a}\times\vec{r}}{r^3}\right) = \frac{3(\vec{a}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{a}}{r^3}.$$ 7

(b) Evaluate :

$$\iint\limits_S \vec{A}\cdot\hat{n}\,dS$$

where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ located in the first octant. 8

6. (a) Evaluate :

$$\oint_C (y - \sin x)\,dx + \cos x\,dy$$

(i) directly

(ii) using Green's theorem in the plane, where C is the boundary of a triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$, and $y = \frac{2}{\pi}x$. 10

Turn over

(b) Verify that :

$$\nabla^2 r^n = n(n+1)r^{n-2}.$$ 5

7. (a) Verify divergence theorem for

$$\vec{F} = \left(x^2 - yz\right)\hat{i} + \left(y^2 - xz\right)\hat{j} + \left(z^2 - xy\right)\hat{k}$$

taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. 10

(b) Express the position and velocity of a particle in cylindrical coordinates. 5

8. (a) Derive an expression for the divergence of a vector field in orthogonal curvilinear coordinate system. 10

(b) Evaluate Jacobian $J\left(\frac{x, y, z}{u_1, u_2, u_3}\right)$ for the transformation from rectangular coordinate system to spherical coordinate system. 5

(20) 2018

This question paper contains **4** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **103**

Unique Paper Code : **32221102** I

Name of the Paper : **Mechanics**

Name of the Course : **B.Sc. (Hons.) Physics (CBCS)**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *five* questions in all.

Q. No. **1** is compulsory.

Use of non-programmable scientific calculator is allowed.

1. Attempt any *five* of the following : 5×3=15

(*a*) Prove that the radius vector sweeps out equal areas in equal intervals of time for any elliptical orbit under central force motion.

(*b*) Explain the theory of expanding universe using Doppler effect in light.

(*c*) What are the effects of Coriolis force due to Earth's rotation.

P.T.O.

(*d*) Show that the ratio of rotational to translational kinetic energy for a solid cylinder rolling down a plane without slipping is 1 : 2.

(*e*) Compare gravitational mass with inertial mass of the body.

(*f*) Show that $E^2 - c^2p^2$ is invariant to Lorentz transformations.

(*g*) Show that damping has little or no effect on the frequency of a harmonic oscillator if its quality factor is large.

(*h*) Explain how a hollow cylinder is stronger than a solid cylinder having same material, mass and length.

2. (*a*) State and prove Work-Energy theorem. 7

(*b*) Show that in an elastic collision of two particles in centre of mass frame of reference, the magnitude of the velocity remains unchanged before and after the collision. 8

3. (*a*) Find the centre of mass of a uniform solid hemisphere of mass M and radius R w.r.t. its geometrical centre. 7

(*b*) Determine the moment of inertia of a uniform hollow sphere of mass M, and radius R about its diameter and tangent. 8

4. (*a*) Derive the expression for the gravitational potential due to a solid sphere of radius R and mass M at a point outside the shell and also at a point inside the shell. 10

(*b*) Show graphically the variation of both gravitational potential and gravitational field as a function of radial distance from the centre of the sphere. 5

5. (*a*) State and prove theorem of perpendicular axes of moment of inertia for a three-dimensional rigid body. 7

(*b*) Establish the relation between Y, K and *n* where Y is the Young's modulus, K is the bulk modulus and *n* is the modulus of rigidity of the material. 8

6. (*a*) Deduce the differential equation of a damped harmonic oscillator and discuss in detail the cases of overdamped, critical and underdamped oscillators. 12

(*b*) A condenser of capacity 1 microF, an inductance of 0.2 Henry and a resistance of 800 ohm are connected in series. Is the circuit oscillatory ? If yes, calculate the frequency and quality factor of the circuit. What do you understand by Quality factor of an oscillator ? 3

7. (*a*) What is Coriolis force ? Show that the total Coriolis force acting on a body of mass m in a rotating frame is $-2m\ \vec{\omega} \times \vec{v}_{rot}$ · where $\vec{\omega}$ is the angular velocity of rotating frame and $\vec{v}_{rot}$ is the velocity of the body in rotating frame. 9

(*b*) Calculate the values of the centrifugal and Coriolis forces on a mass of 20 g placed at a distance of 10 cm from the axis of a rotating frame of reference, if the angular speed of rotation of the frame be 10 radians per second. 4

(*c*) Calculate the effective weight of an astronaut ordinarily weighing 60 kg when his rocket moves vertically upward with 5 g acceleration. 2

8. (*a*) Describe Michelson-Morley experiment and explain the significance of the null result. State the postulates of special theory of relativity. 6,2,2

(*b*) The proper mean life time of pi meson is 2.5×10^{-8} sec. Calculate :

(i) the mean life time of pi meson travelling with velocity 2.4×10^{10} cm/sec.

(*ii*) distance travelled by it before disintegrating. 3,2